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Semileptonic Decays of Vector Mesons $[\rho, \omega, \phi] \rightarrow \pi[e^+e^-, \mu^+\mu^-]$ in a Chiral NJL Model

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Abstract—Semileptonic decays of vector mesons ρ , ω , ϕ into a pion and lepton pair e^+e^- , $\mu^+\mu^-$ are described in the chiral Nambu–Jona–Lasinio (NJL) model. Mixing of ω with ϕ is taken into account in a description of vector ϕ meson decay. Direct production via a photon field and production with the participation of intermediate vector mesons ρ or ω are taken into account in the production of l^+l^- lepton pairs. The total and differential widths are compared with an experiment. Predictions for $\rho \rightarrow \pi[e^+e^-, \mu^+\mu^-]$ and $\phi \rightarrow \pi\mu^+\mu^-$ decays are also given.

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INTRODUCTION

The investigation of lepton pair production in decays of light vector mesons has received new attention in recent years. Investigating these decays is a useful tool for a deeper understanding of the structure of mesons. These processes have been extensively studied during many years both theoretically and experimentally [1, 2].

Theoretical studies of the processes of $V \rightarrow \pi l^+ l^-$ were performed earlier in the framework of the vector dominance model in [3, 4]. The processes of interest were also studied in [5] using Hidden Local Symmetry (HLS) Lagrangian. Decays $\omega \rightarrow \pi l^+ l^-$ were described in recent papers [6, 7] using transition form factors in good agreement with experiment.

It is interesting to note that the problem of correctly describing the ratio of decays $\Gamma(V \rightarrow \pi\mu^+\mu^-)/\Gamma(V \rightarrow \pi e^+e^-)$ arises in many theoretical studies; in theory, this ratio turns out much smaller than in experiment. A similar situation, but with a substantial excess of the theoretical prediction over experimental value, takes place in investigation of decays of heavy mesons into a kaon and a lepton pair $B \rightarrow K l^+ l^-$ [8–10]. This indicates the possible existence of close common sources of this phenomenon both for heavy and light mesons.

In this study, we calculate the above processes based on effective chiral Lagrangians obtained in the well-known Nambu–Jona–Lasinio (NJL) model [11–19]. Along with a comparison with available data, we give predictions for the total and differential decay widths for $\rho \rightarrow \pi^0 l^+ l^-$ and $\phi \rightarrow \pi^0 l^+ l^-$. The results can be verified in future experiments. Note that the accuracy of the model calculations is determined by the accuracy of conservation of chiral symmetry, which is the basis of the NJL model. This accuracy depends on conservation of axial current (principle of partial conservation of the axial current, PCAC) and is estimated as $\pm 10\%$.

INTERACTION LAGRANGIAN OF THE NJL MODEL

The Lagrangian of the NJL model describing the interaction of photons, pions, and vector mesons with quarks has the form [12]

$$\begin{aligned} \Delta L = & \bar{q} \left[i\gamma_\mu \partial_\mu - m + \frac{e}{2} \left(\tau_3 + \frac{1}{3} I \right) \gamma_\mu A_\mu \right. \\ & + i g_\pi \gamma_5 \tau_3 \pi^0 + \frac{g_\rho}{2} \tau_3 \gamma_\mu \rho_\mu^0 \\ & \left. + \frac{g_\rho}{2} I (\cos(\alpha) \gamma_\mu \omega_\mu + \sin(\alpha) \gamma_\mu \phi_\mu) \right] q, \end{aligned} \quad (1)$$

Fig. 1. Contact diagram.

where q and $\bar{q}$ are the fields of u and d quarks with the constituent masses $m_u \approx m_d = 280$ MeV [12], $m = \text{diag}(m_u, m_d)$; e is the electron charge; A is the photon field; $\rho, \omega, \phi, \pi^0$ are the meson fields; τ_3 is the Pauli matrix; and I is the identity matrix. The mixing angle for ω and ϕ mesons is $\alpha = 3.1^\circ$ [20].

The coupling constants are expressed in terms of logarithmically divergent integrals:

$$g_\rho = \sqrt{\frac{3}{2I_2}} \approx 6.14, \quad g_\pi = \sqrt{\frac{Z_\pi}{4I_2}} \approx 3.02, \quad (2)$$

where

$$Z_\pi = \left(1 - 6 \frac{m^2}{M_{a_1}^2}\right)^{-1} \approx 1.4. \quad (3)$$

Here, Z_π is an additional renormalization constant that appears in $\pi - a_1$ transitions; $M_{a_1} = 1230 \pm 40$ MeV is the meson $a_1(1260)$ mass [21].

The integral in quark loops has the form

$$\begin{aligned} I_2 &= -i \frac{N_c}{(2\pi)^4} \int \frac{\Theta(\Lambda_4^2 + k^2)}{(m^2 - k^2)^2} d^4k \\ &= \frac{N_c}{(4\pi)^2} \left[\ln \left(1 + \frac{\Lambda_4^2}{m^2}\right) + \frac{\Lambda_4^2}{\Lambda_4^2 + m^2} \right], \end{aligned} \quad (4)$$

where $\Lambda_4 = 1.26$ GeV is the four-dimensional cut-off parameter [12]; $N_c = 3$ is the number of colors in QCD. This integral appears after the expansion of the loop integrals over the external momentum and handling logarithmically divergent integrals. This is the only way to conserve a chirally symmetric form of the Lagrangian [12]. The minimal order with respect to external momenta is taken into account in calculating anomalous diagrams.

$[\omega, \rho, \phi] \rightarrow \pi [e^+ e^-, \mu^+ \mu^-]$ DECAYS

The processes $[\omega, \rho, \phi] \rightarrow \pi [e^+ e^-, \mu^+ \mu^-]$ in the NJL model are described by the contributions of two dia-

Fig. 2. Diagram with an intermediate vector meson.

grams: the triangular quark diagram corresponding to direct production of lepton pairs via the photon field (Fig. 1) and the diagram with an intermediate vector meson connecting the anomalous triangular diagram with the vertex of the transition $(\rho, \omega) \rightarrow \gamma \rightarrow l^+ l^-$ (Fig. 2). The calculations give the following amplitude:

$$\begin{aligned} \mathcal{M} &= \frac{N_c g_\rho \alpha_{em} C_{VP}}{2\pi F_\pi} \frac{1}{s} \left[1 + \frac{s}{M_V^2 - s - i\sqrt{s}\Gamma_V} \right] \\ &\times \varepsilon_{\mu\nu\lambda\delta} e_\mu^*(p) L_\nu p_{0\lambda} p_\delta, \end{aligned} \quad (5)$$

where F_π is the constant of weak pion decay; p is the vector meson momentum; p_0 is the pion momentum; $e_\mu^*(p)$ is the polarization vector of the vector meson; $s = (p - p_0)^2 = (p_+ + p_-)^2$; $L_\nu = \bar{l} \gamma_\nu l$ is the lepton current; and the constants $C_{\omega\pi} = \cos(\alpha)$, $C_{\phi\pi} = \sin(\alpha)$, and $C_{\rho\pi} = 1/3$. M_V and Γ_V are the mass and width of the vector meson in the intermediate state.

NUMERICAL RESULTS

The decay probability for $V(p) \rightarrow \pi(p_0) + l^+(p_+) + l^-(p_-)$ can be calculated using the formula

$$\Gamma(V \rightarrow \pi l^+ l^-) = \frac{1}{3} \frac{1}{256\pi^3 M_V^3} \int_{s_-}^{s_+} ds \int_{t_-(s)}^{t_+(s)} dt |\mathcal{M}|^2, \quad (6)$$

where the variables are defined as follows: $s = (p - p_0)^2 = (p_+ + p_-)^2$ and $t = (p - p_+)^2 = (p_0 + p_-)^2$. The integration limits have the form

$$s_+ = (M_V - M_\pi)^2, \quad s_- = 4M_l^2, \quad (7)$$

$$\begin{aligned} t_\pm(s) &= \frac{1}{2} [M_V^2 + M_\pi^2 + 2M_l^2 - s \\ &\pm s^{-1} \sqrt{\lambda(s, M_l^2, M_l^2)} \lambda(s, M_V^2, M_\pi^2)], \end{aligned} \quad (8)$$

where $\lambda(a, b, c) = (a - b - c)^2 - 4bc$.

The results and the experimentally measured decay widths are given in Tables 1–3. The calculated differential distributions of the studied processes over the

Table 1. $V \rightarrow \pi l^+ l^-$ decay widths

Decays	NJL, keV	[7], keV	[6], keV	[5], keV	PDG [21], keV
$\omega \rightarrow \pi e^+ e^-$	6.81	6.67 ± 0.21	6.93 ± 0.09	6.68 ± 0.2	6.5 ± 0.5
$\omega \rightarrow \pi \mu^+ \mu^-$	0.61	0.82 ± 0.02	0.98 ± 0.05	0.67 ± 0.02	1.10 ± 0.15
$\rho \rightarrow \pi e^+ e^-$	0.78	—	—	0.91 ± 0.08	< 1.7
$\rho \rightarrow \pi \mu^+ \mu^-$	0.058	—	—	0.086 ± 0.007	—
$\phi \rightarrow \pi e^+ e^-$	0.063	0.062 ± 0.002	—	0.064 ± 0.004	—
$\phi \rightarrow \pi \mu^+ \mu^-$	0.013	0.016 ± 0.001	—	0.013	—

invariant mass of the lepton pair $l^+ l^-$ are shown in Figs. 3–8.

DISCUSSION AND CONCLUSIONS

The first estimates for $\omega \rightarrow \pi \mu^+ \mu^-$ decay were made in [3, 4] using the vector dominance method. These papers described transition form factors depending on momentum of the intermediate photon beyond the mass surface. At the same time, all decays of interest were studied in the framework of the HLS model [5]. Moreover, the processes $\omega \rightarrow \pi l^+ l^-$ were studied in recent papers [6, 7]; a model based on the chiral perturbation theory was used in one of the papers, while the other one applied the method of dispersion relations. All this demonstrates the interest in investigating the considered processes from the theoretical and experimental points. That is why we thought it interesting to try to describe all these processes in the framework of the chiral NJL model,

which so far has provided satisfactory results for many processes in the low-energy region [12].

Note that the vector dominance method used in a number of the abovementioned theoretical studies [3, 4] is automatically reproduced in the NJL model. Vector dominance appears in the chiral NJL model when the two channels in which the considered processes happen are united. The first is connected with an intermediate photon producing a lepton pair. The second channel is connected with an intermediate vector meson transformed into a photon and also producing a lepton pair. The independent examination of these channels ensures a more detailed study of the processes of interest. We also calculated the differential widths of all the decays described above.

An examination of separate channels yielded an interesting result. It turned out that the dominant channel in the production of the electron–positron pair is the channel with an isolated photon which completely defines the width of these processes. The vector channel with the participation of an intermediate ρ or ω meson gives an insignificant correction to the decay width. A different situation takes place in the

Fig. 3. Differential width of the process $\omega \rightarrow \pi \mu^+ \mu^-$. Experimental points were taken from [2].

Fig. 4. Differential width of the process $\omega \rightarrow \pi e^+ e^-$.

Fig. 5. Differential width of the process $\phi \rightarrow \pi\mu^+\mu^-$. Fig. 6. Differential width of the process $\phi \rightarrow \pi e^+e^-$. Fig. 7. Differential width of the process $\rho \rightarrow \pi\mu^+\mu^-$. Fig. 8. Differential width of the process $\rho \rightarrow \pi e^+e^-$.

production of a more massive $\mu^+\mu^-$ pair. Here, the dominant role is also that of the single-photon channel; however, the contribution of the vector channel with intermediate ρ or ω meson is more important. Thus, for example, in $\omega \rightarrow \pi\mu^+\mu^-$ decay, taking into account the photon channel alone yields the width $\Gamma(\omega \rightarrow \pi\mu^+\mu^-) = 0.3$ keV, while the addition of the vector channel with ρ meson yields $\Gamma(\omega \rightarrow \pi\mu^+\mu^-) = 0.61$ keV. A similar situation takes place in $(\rho, \phi) \rightarrow \pi\mu^+\mu^-$ decays as well.

A positive interference between the contributions of the contact diagram and the diagram with an intermediate vector meson was found for all described decays. Note that this situation differs substantially from the processes of hadron production on colliding

l^+l^- beams. Indeed, in this case, the channel with an isolated photon is, as a rule, in negative interference with the vector channel containing a ρ or ω meson in the intermediate state. The latter channel plays a dominant role [22, 23].

It is also interesting to note that the possibility of $\phi \rightarrow \pi l^+l^-$ decays is explained by the mixing of ω and ϕ mesons with the angle $\alpha = 3.1^\circ$ [20]. The ϕ meson decay widths turned out to be close to the results obtained in [5, 7] (see Table 1).

The results obtained using the model can be regarded as sufficiently satisfactory, except for $\omega \rightarrow \pi\mu^+\mu^-$ decay. Here, unfortunately, it is impossible to achieve a satisfactory agreement with the experiment in the framework of the model. As was already

mentioned in the Introduction, a similar situation takes place in the investigation of decays of heavier mesons $B \rightarrow K\ell^+\ell^-$.

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